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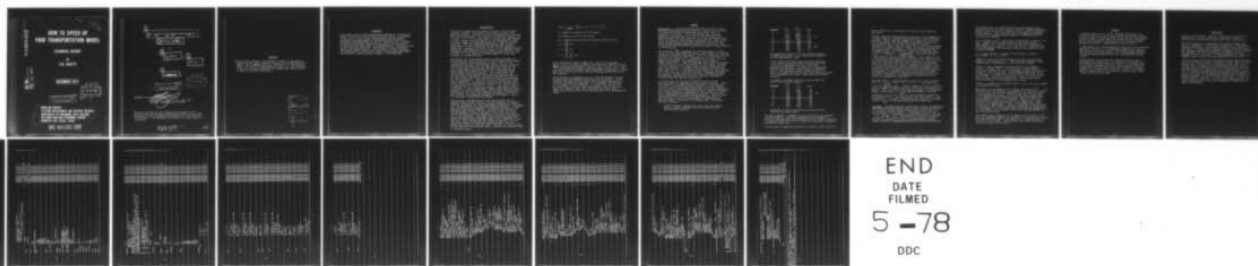
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HOW TO SPEED UP YOUR TRANSPORTATION MODEL. (U)
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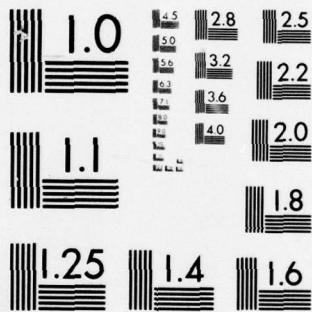
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HOW TO SPEED UP YOUR TRANSPORTATION MODEL

TECHNICAL REPORT

BY
T.M. BEATTY

DECEMBER 1977

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MODELING BRANCH
SYSTEMS DEVELOPMENT AND SUPPORT DIVISION
DIRECTORATE OF PERSONNEL DATA SYSTEMS
AIR FORCE MILITARY PERSONNEL CENTER
RANDOLPH AFB, TEXAS 78148

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⑥

HOW TO SPEED UP YOUR TRANSPORTATION MODEL.

⑨

TECHNICAL REPORT

BY

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T.M. BEATTY

⑫

22 p.

⑪

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APPROVED BY:

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FREDERICK S. PHILLIPS, SR., COL, USAF
DIRECTOR, PERSONNEL DATA SYSTEMS

While the contents of this report are considered to be correct, they are subject to modification upon further study. This report does not promulgate official Air Force policies or positions. The technical conclusions are solely those of the author.

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ABSTRACT

✓ This report presents a new method to compute a more nearly optimal initial basic feasible solution for the Transportation Model. The integration of two techniques; (1) The Decision Index (DI) and (2) An Admissability Index (AI), have resulted in 50 to 75 percent reductions in computer run time required to derive an optimal solution. ↑

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FOREWORD

This report and the BEST program were prepared by the Modeling Branch of the Air Force Military Personnel Center in response to the need to solve large transportation and assignment problems in the management of the approximately one million personnel employed by the Air Force. The work of Dr. Joe Ward of the Air Force Human Resources Laboratory, who originally authored The Decision Index, is acknowledged for its contribution to the technique presented. Additionally, Gloria Jeaneatte McWilliams, whose thesis at the University of Texas indicated the efficiency of the Decision Index in developing approximate solutions for transportation problems, is acknowledged.

INTRODUCTION

To solve a transportation problem you need an initial basic feasible solution. The better (more nearly optimal) the starting solution, the more rapid the transportation algorithm will go to convergence. As so succinctly stated by Hadley (#1) in 1962, "-----", It is worth while to spend some time finding a 'good' initial solution because it can considerably reduce the total number of iterations required to reach an optimal solution." For any transportation problem there exists a continuum of initial solutions ranging from the Northwest Corner Rule (probably the worst) to one of the numerous optimal solutions. The objective of this report is to describe a new technique which derives initial solutions which are very near optimum, i.e. will require a reduced number of iterations to reach an optimal solution.

McWilliams (#2) investigated several promising methods for developing approximate solutions to the transportation problem in her thesis at the University of Texas. In brief her conclusions dismissed the Northwest Corner Rule as a serious alternative and found that the Ward Decision Index and the C^Q , in both static and dynamic versions, were superior to other algorithms currently in use, e.g. row/column minima, cost minima, and Vogel's method. She further noted that Ward's method and C^Q method differ only by additive and multiplicative constants in the static version when $M=N$ and for this reason the initial feasible solutions are identical for the two methods. (But this is only true when $M=N$ in the static versions). It should be noted that McWilliams' thesis used a slight misinterpretation of Ward's DI. The original DI computation (#3) involves $C_{.j}$ and $C_{i.}$, the sums of the j^{th} column and i^{th} row of the cost tableau. These sums consider the quotas in their computation. McWilliams' version of the DI computes the row and column sums without quota consideration. Ward suggest (#4) that this incorrect computation of the DI may not have significantly degraded the usefulness of the DI for starting a transportation problem.

The original promulgation of DI was made by Ward (#3) in seeking a near optimal solution to the sequential assignment problem confronting Air Force assignment counselors in dealing with job assignments for non-prior service personnel. Counselors were required to make sequential assignment decisions in the absence of perfect information. As each airman arrived at the assignment station, he was given his initial Air Force career assignment without knowledge of the vast labor pool yet to be considered. The result was intuitively sub-optimal where each sequential assignment subjectively maximized the payoff criteria against whatever happen to be remaining in the job pool. Ward conjectured that given estimates of the row and column means, of admittedly imprecise data, that the sequential assignment problem could be solved in a near optimum manner employing the Decision Index. Wherein the Decision Index is computed thusly:

$$DI_{ij} = \frac{1}{N(M-1)} (MC_{ij} - C_{i.} - C_{.j} + C_{..})$$

M = Number of persons to be assigned,

N = Number of jobs to be filled,

C_{ij} = Productivity of the i th person on the j th job,

$$C_{i.} = \sum_{j=1}^N C_{ij}$$

$$C_{.j} = \sum_{i=1}^M C_{ij}$$

$$C_{..} = \sum_{i=1}^M \sum_{j=1}^N C_{ij}$$

As is discussed in Ward's paper (#3), this DI is the mean value of the cost used to compute the objective function for all $\frac{(M-1)!}{(M-N)!}$ possible assignments of the i th man to the j th job. (Note: This seemingly powerful technique has, to the best of this authors knowledge, not been widely implemented either as an assignment tool or in conjunction with the Transportation Model as is now proposed).

Early attempts to implement DIs as a starting solution for the transportation/assignment problem were not successful. These failures resulted from the type of problem being examined in which M, the number of personnel to be assigned, far exceeded N, the number of jobs available. In these first efforts, it was found that a DI start was little better, and in fact more costly in computer time, than a modified row minima start. These early failures led to the consideration of an admissability index.

METHOD

Background: A straight-forward application of DI's to the personnel assignment problem where $M > N$ resulted in solutions not much better than the modified row minima. This, in hindsight, is obvious. If all contenders were at least marginally qualified for all jobs in the case where 2000 persons sought 300 jobs, then the first 300 contenders would enter the solution and remaining 1700 would not. This result could be marginally better than a modified row minima but would consume greater computer resources in deriving. Thus the need for some method of selecting the order of admissability to the solution.

The statistic, $\frac{\mu_{ci}}{\sigma_{ci}}$, possessed an intuitive appeal and has proven a useful admissability index in application to personnel assignment problems investigated. Here μ_{ci} represents the mean value of the costs for the i th person for all real jobs (in application a shadow job is used which has a quota of M , the number of personnel to be assigned and is not used in computing μ_{ci} and σ_{ci}). The σ_{ci} represents the standard deviation of the costs for the i th person for all real jobs. The mean cost for a person is an overt index of which personnel will enter the solution. In general those persons with smallest mean costs will be in the solution and those with the greatest mean cost will not. Inclusion of σ_{ci} as a moderator provides for earlier introduction to the solution of those who might be otherwise unattractive but have a few good assignment possibilities and limited alternatives.

In calculating the statistics μ_{ci} and σ_{ci} as well as the Decision Index, an important deviation was introduced. BIG M was observed to possess dramatic influence in the starting solution. (BIG M originally generated in the cost tableau is two orders of magnitude greater than the largest admissible real cost.) Early on, it was observed that using the original BIG M caused marginal assignments in the starting solution which subsequently left the optimal solution. This of course results from the use of σ_{ci} in the admissability index and the follow-on use of the column means in computing the Decision Index. Reference to an example best illustrates why this occurred.

Given 10 person competing for five jobs with admissability indices computed using the original BIG M, the following results:

| Person# | μ_{ci} | σ_{ci} | AI | |
|---------|------------|---------------|-----|-----|
| 1 | 200 | 1000 | .20 | |
| 2 | 210 | 1050 | .20 | |
| 3 | 1100 | 4400 | .25 | IN |
| 4 | 350 | 1166 | .30 | |
| 5 | 300 | 750 | .40 | |
| 6 | 400 | 800 | .50 | |
| 7 | 450 | 818 | .55 | |
| 8 | 460 | 767 | .60 | OUT |
| 9 | 500 | 769 | .65 | |
| 10 | 550 | 786 | .70 | |

The moderating effect of σ_{ci} has effected exactly as intended and persons #3 and 4 were driven upward on the admissability list.

Given further that there was one highly difficult job in the quota bank on which most personnel had a BIG M cost and person #3 had a high cost, marginally acceptable. The resulting large column mean for the difficult job when used in computing the DI (see discussion below) caused person #3 to have an attractive DI and to enter the starting solution on an assignment that later must leave the optimal solution.

If however a suitable substitution were made for BIG M, the following results:

| Person# | μ_{ci} | σ_{ci} | AI | |
|---------|------------|---------------|-----|----|
| 1 | 200 | 1000 | .20 | |
| 2 | 210 | 1050 | .20 | |
| 5 | 300 | 750 | .40 | IN |
| 6 | 400 | 800 | .50 | |
| 4 | 340 | 654 | .52 | |
| 7 | 450 | 818 | .55 | |
| 8 | 460 | 767 | .60 | |
| 9 | 500 | 769 | .65 | |
| 10 | 550 | 786 | .70 | |
| 3 | 800 | 1111 | .72 | |

Note that #3 leaves the starting solution and #4 remains but enters later.

The above example, intended only as illustrative and probably not derivable with real data, demonstrates an anomaly of Decision Indexing noted by and currently under study by Ward(#5) A rigorous derivation of how to handle this problem is expected from Ward's study (#5).

In this work, an empirical derivation of BIG M = 1025 with real

costs ≤ 998 and > 0 was found to result in good starting solutions.

Another important consideration is the cost of the shadow job. In early usage of transportation modeling to solve personnel problems, shadow jobs were costed at a fixed cost, less than BIG M but greater than the greatest admissible actual cost. The rationale was, of course, to assign all eligible contenders to actual jobs and assure that no BIG M assignments were in the optimal solution. When this shadow cost was used in computing DIs, the shadow DIs did not discriminate over contenders since all DIs in the shadow job column were equal to zero, i.e. $[C_{\text{shadow}} - \frac{(C_{\text{shadow}}) * M}{M}]$. Again the statistic

$\frac{P_{ci}}{C_{ci}}$ (the AI) has an intuitive appeal. Since there will be no contention concerning people with small AIs, they will be in the solution and likewise, the large AIs will not be in the solution, the cost index for shadow jobs must discriminate about the threshold just prior to exhausting the quota bank. At this point there are perhaps contenders who should not be included in the initial start but should be assigned to the shadow job while the search continues for more cost-effective assignments. In several test runs, the AI used as shadow cost has demonstrated the required discrimination about the threshold. Therefore, from empirical considerations only, the AI is proposed for this role.

Decision Indices, as promulgated by Ward (#3), were computed with $DI_{ij} = \frac{1}{N(M-1)} (MC_{ij} - C_i - C_j + C_{..})$ (see page 5

above for amplification). Several elements of this calculation are not needed in computing DI for starting a Transportation Model. Since the initial assignments are made in a sequential manner, the division by $\frac{1}{N(M-1)}$ may be deleted. Also the

subtraction of the row total, C_i , may be removed as well as the addition of $C_{..}$, the total sum of the cost tableau. This leaves $MC_{ij} - C_j$ which is computationally easier to implement as $C_{ij} - \frac{C_j}{M}$, or the deviation from the column mean.

Software: The BEST program is presented in ALGOL in Appendix A. The various computations executed in BEST to produce the basis for Langley's Primal Simplex Transportation Model (#6) will be documented in this section. As appropriate, reference will be made to line numbers in the program listing. In line 300 through 4300, declarations of files and variables are found. Only three of the files are pertinent to the general application of starting a transportation problem. These are INP1 or CAREERS/DATA/TRANS which provides the number

of destinations, the number of sources, the demand at each destination, and the cost tableau. ADVAN or CAREERS/DATA/BASIS which is used to write the basis for later input to Langley's Primal Simplex. The third file DIJS or CAREERS/DATA/REVERSE is used as a transient storage medium in computing the DI's from the cost tableau.

At line number 4400, the size of the problem is read in from INPl. These data are then printed in the output report. Line numbers 5000 through 10200 provide more declarations. Three defines are shown at 10300 to 10600. These are used to bit pack/unpack the variable A which is tag sorted in the procedure SORTER.

An in place tag sort is shown in the procedure SORTER at line number 10700 to 23900.

Demands at each of the destinations are read into the array NJOBS at line number 24100. Then the total number of jobs to be allocated is accumulated in the variable, TOTJOBS.

Costs are read in sequentially from INPl at line numbers 26500 through 30800. In this block of code the BIG M adjustment is made and then the necessary computations for computing N_{ci} and C_{ci} are made. From these calculations the AI is computed, bit packed with the row index, and stored in the array ROWTOT. The AI is also stored in cost (Ø) to be used later as the cost for the shadow job. The adjusted costs are then written to the transient file, DIJS.

SORTER is invoked at line number 30900 to sort the array ROWTOT on ascending AI value.

Beginning at line number 32300 through line number 38700, the adjusted costs are randomly read (based on the sorted AI) from the file DIJS, the DI's are computed, and the basis is stored in the appropriate arrays. Of particular note in this area is the decrementing and testing on the variable TOTJOBS which is the number of jobs to be allocated. When all jobs have been allocated, this area is left and a wrapup area is used to assign all remaining personnel to the Shadow Job. Also note at line number 34000, the DI is computed dynamically as a function of the number of personnel remaining to be assigned. To accomplish this, the column totals are adjusted appropriately at line number 38400 through 38500 after each assignment in the starting solution.

The wrapup area is shown at line number 40200 through 45200. Here the necessary elements of the basis are calculated to assign all remaining personnel to the Shadow Job. Then the basis is written to file, ADVAN. Other reports and timing statistics are also written at this point.

RESULTS

A typical application of AI/DI starting technique is its production usage in the Air Force CAREERS JOB FINDER System. In this system, first-term airman who do not have reenlistment quotas in their current AFSCs are costed against jobs which do not have sufficient applicants to meet Air Force career manpower objectives. This produces a cost tableau.

In the problem run, 1908 prospective reenlistees were optimized against 939 jobs in 79 job categories producing an implicit cost array of 1,791,612 cells. The explicit array contained 150,732 cells with 100% density, i.e. inadmissible person-job-matches were costed at BIG M but were defined to Primal Simplex.

Primal Simplex using Langley's internal artificial start converged to a solution in 1,643 seconds of processor time. The problem was then solved using the AI/DI to generate the initial basic feasible solution. This required 87 seconds to generate the basis and 306 seconds in Langley's Primal Simplex to go to an optimal solution. That is, the problem takes 4 times longer to run with the artificial start than it does with the AI/DI start.

CONCLUSION

Hadley's truism stands. Ward's contribution to the arena of optimization is again recognized. McWilliam's initial investigation has served to stimulate further investigation.

The AI/DI technique is implemented with the prototype code shown at Appendix A. Results are satisfactory. Large person-job-match problems are now taken in stride without regard to ADP impact. In fact, these problems consume so little time that the stringent limitations on memory utilization can be relaxed. This relieves the user of the onerous problems of memory management which may prevail in some installations. There are, however, areas in which further research may be fruitful.

Given that many optimization problems deal with imprecise data, even to the extent of being subjective in some cases, how near optimal must a solution be, to be good enough? Does the AI coupled with a dynamic DI get close enough for most, some, or any optimization problems? Can tests be applied to the AI/DI objective function to determine if it is good enough, i.e. 1, 2, 3, or perhaps 22 standard deviations from the mean objective value? Can the AI/DI be applied to sparse matrix problems? Network problems? Is the documented computation of the AI the correct method? The best? or only one of many? Is there a better index than the AI to use for costing Shadow Jobs? Is there adequate payoff from the introduction of Shadow Jobs to make it worthwhile?

REFERENCES

1. Hadley, G., Linear Programming. Addison-Wesley, Reading, MA, 1962.
2. McWilliams, G.J., A Method for the Approximate Solution of Transportation Problems. Austin, TX, 1970.
3. Ward, Joe H. Jr., "The Counseling Assignment Problem," Psychometrika, Vol 23, #1, March 1958.
4. ---. Memo for the Record, Subj: Comments on " A Method for the Approximate Solution to Transportation Problems" by McWilliams. San Antonio, TX, October 1974.
5. ---. Informal Communications, 1974-1977.
6. Langley, R.W. et al: Efficient Computational Devices for the Capacitated Transportation Problem. United States Air Force Academy, 1975.

BEST

```

$SET MERGE
$ SET LIST FORMAT LINEINFO
BEGIN
    00000100 000:0000:0
    00000200 000:0000:0
    00000300 003:0000:1
    00000400 003:0000:1
    00000500 003:0000:1
    DATA IS 000C LONG
    00000600 003:0000:1
    00000700 003:0000:1
    00000800 003:0000:1
    00000900 003:0000:1
    DATA IS 000D LONG
    00001000 003:0000:1
    00001100 003:0000:1
    DATA IS 0005 LONG
    00001200 003:0000:1
    00001300 003:0000:1
    DATA IS 0005 LONG
    00001400 003:0000:1
    00001500 003:0000:1
    00001600 003:0000:1
    DATA IS 000A LONG
    00001700 003:0000:1
    00001800 003:0000:1
    00001900 003:0000:1
    DATA IS 000B LONG
    00002000 003:0000:1
    00002100 003:0000:1
    00002200 003:0000:1
    DATA IS 000R LONG
    00002300 003:0000:1
    00002400 003:0000:1
    00002500 003:0000:1
    00002600 003:0000:1
    00002700 003:0000:1
    00002800 003:0000:1
    00002900 003:0000:1
    00003000 003:0000:1
    00003100 003:0000:1
    00003200 003:0000:1
    00003300 003:0000:1
    00003400 003:0000:1
    00003500 003:0000:1
    00003600 003:0000:1
    00003700 003:0000:1
    00003800 003:0000:1
    00003900 003:0000:1
    00004000 003:0000:1
    00004100 003:0000:1
    00004200 003:0000:1
    00004300 003:0000:1

FILE
    DIJS(KIND=DISKPACK,PACKNAME="DADATA.",TITLE="CAREERS/DATA/R",
    "EVERSE.",MYUSE=10,UNITS=WORDS,AREASIZE=100,AREAS=70);

FILE
    QTP(KIND=DISKPACK,PACKNAME="DADATA.",TITLE="CAREERS/DATA/DI",
    "OFFR.",MAXRECSIZE=2,BLOCKSIZE=600,UNITS=WORDS,
    AREASIZE=1200,AREAS=10);

FILE
    CARD(KIND=READER);

FILE
    PRIN(KIND=PRINTER);

FILE
    INP1(KIND=DISKPACK,PACKNAME="DADATA.",TITLE="CAREERS/DATA/T",
    "RANS.",FILETYPE=7);

FILE
    INP2(KIND=DISKPACK,PACKNAME="DADATA.",MYUSE=10,
    TITLE="CAREERS/DATA/INVERSE.",FILETYPE=7);

FILE
    ADVAN(KIND=DISKPACK,PACKNAME="DADATA.",TITLE="CAREERS/DATA/",
    "BASIS.",AREASIZE=1,AREAS=1,FILETYPE=7);

INTEGER
    I,
    J,
    K,
    NROWS,
    NCOLS,
    COUNT,
    INIT,
    ISAV;

REAL
    NASSN,
    DEC;

REAL
    BIGH,
    SUM,
    BUF,
    OBJVAL;

FORMAT
    F5(X5,16,X4,16,X4,110,X4,110);

FORMAT
    F6(X6,13,X6,13);
    
```

APPENDIX A

| | | |
|---|-------------------------|------------|
| READ(INP101,<216>,NCOLS,NROWS); | 00004100 | 003:0000:1 |
| DLJS,MAXRECSIZE:=NCOLS; | 00004500 | 003:000A:2 |
| NASSN:=NROWS; | 00004600 | 003:000C:2 |
| WRITE(PRIN,<" NR. AFSC,S(COLS)= ",16, | 00004700 | 003:000D:1 |
| " NR. AIRMEN(ROWS)= ",16>,NCOLS,NROWS); | 00004800 | 003:000F:0 |
| BEGIN | DATA IS 0013 LONG | |
| REAL | 00004900 | 003:0015:2 |
| TOTJOBS; | 00005000 | 003:0015:2 |
| | 00005100 | 003:0015:2 |
| REAL | B.0001 IS SEGMENT 00008 | |
| LL, | 00005200 | 008:0000:1 |
| MM; | 00005300 | 008:0000:1 |
| REAL ARRAY POINT10:NCOLS1; | 00005400 | 008:0000:1 |
| REAL | 00005500 | 008:0000:1 |
| TIME1, | 00005600 | 008:0003:1 |
| TIME2, | 00005700 | 008:0003:1 |
| TIME3, | 00005800 | 008:0003:1 |
| TIME4, | 00005900 | 008:0003:1 |
| ROWSUM, | 00006000 | 008:0003:1 |
| COLTEM; | 00006100 | 008:0003:1 |
| REAL | 00006200 | 008:0003:1 |
| TIMER, | 00006300 | 008:0003:1 |
| JOBS; | 00006400 | 008:0003:1 |
| REAL ARRAY ID, | 00006500 | 008:0003:1 |
| IJ, | 00006600 | 008:0003:1 |
| IR, | 00006700 | 008:0003:1 |
| IP, | 00006800 | 008:0003:1 |
| IDA(C:NROWS+NCOLS1); | 00006900 | 008:0003:1 |
| LABEL | 00007000 | 008:0003:1 |
| START; | 00007100 | 008:0008:4 |
| WRAP; | 00007200 | 008:0008:4 |
| REAL ARRAY NJOB10:NROWS-11, | 00007300 | 008:0008:4 |
| QUANT10:NROWS1, | 00007400 | 008:0008:4 |
| OFFERS10:NROWS, 1:31; | 00007500 | 008:0008:4 |
| REAL ARRAY EXCESS10:NCOLS1; | 00007600 | 008:000C:0 |
| REAL ARRAY ROW10:10:NROWS-11, | 00007700 | 008:000F:0 |
| COST, | 00007800 | 008:0012:3 |
| COLS10, | 00007900 | 008:0015:3 |
| COL, | 00008000 | 008:0018:5 |
| COLS10:10:NCOLS-11; | 00008100 | 008:0018:5 |
| REAL | 00008200 | 008:0018:5 |
| DI; | 00008300 | 008:0018:5 |
| KOS1J; | 00008400 | 008:001D:4 |
| UP; | 00008500 | 008:001D:4 |
| INTEGER | 00008600 | 008:001D:4 |
| INTEGER | 00008700 | 008:001D:4 |
| Z; | 00008800 | 008:001D:4 |
| LABEL | 00008900 | 008:001D:4 |
| OUT; | 00009000 | 008:001D:4 |
| INTEGER | 00009100 | 008:001D:4 |
| KUP, | 00009200 | 008:001D:4 |
| KLO; | 00009300 | 008:001D:4 |
| REAL | 00009400 | 008:001D:4 |
| ROWSQ; | 00009500 | 008:001D:4 |
| REAL | 00009600 | 008:001D:4 |
| ROWTEM; | 00009700 | 008:001D:4 |
| REAL | 00009800 | 008:001D:4 |
| REAL | 00009900 | 008:001D:4 |
| REAL | 00010000 | 008:001D:4 |
| REAL | 00010100 | 008:001D:4 |

```

DEFINE DIK;
PACK(A,B) = A := (A)6(B)[47:13:14] #,
UNPACKDI(A) = (A).[33:34] #,
UNPACKJ(A) = (A).[47:14] #;

PROCEDURE SORTER(A,IB,IC,FB,BC);
  X IN PLACE TAG SORT CACH ALGORITHM #347
  X THIS SORT ROUTINE HANDLES A ONE DIMENSIONAL ARRAY. IT IS MODIFIED
  X TO USE ONLY A SUBSET OF THE BITS OF THE ELEMENTS AS THE KEY. IN
  X THIS MANNER THE ONE WORD MAY CARRY BOTH THE KEY AND 'TAG'. THE
  X FORMAL PARAMETERS OF THE PROCEDURE ARE:
  X
  X A := ONE DIMENSIONAL DATA ARRAY
  X IB := LOW INDEX OF DATA
  X IC := HIGH INDEX OF DATA
  X FB := FIRST BIT OF PARTIAL WORD OF SORT KEY
  X BC := NUMBER OF BITS IN THE KEY
  X
  ARRAY A[*];

  INTEGER IB,
  IC,
  FB,
  BC;

  BEGIN
    IUT0:161,
    ILI0:161;

    REAL T,
    TT;

    INTEGER I,
    J,
    K,
    L,
    M,
    IJ;

    LABEL LB,
    L10,
    L20,
    L30,
    L40,
    L60,
    L60,
    L70,
    L80,
    L90,
    L100,
    EXIT;

    DEFINE KEY
      M := 1;
      I := IB;
      J := IC;

      IF I GEQ J THEN
        L5:

```

| | | | |
|------|-----------------------------|----------|------------|
| L10: | GO TO L70; | 00016100 | 009:0003:4 |
| | K := I; | 00016200 | 009:0004:1 |
| | IJ := (J+1)/2; | 00016300 | 009:0004:1 |
| | T := A(IJ); | 00016400 | 009:0005:1 |
| | IF ALL KEY LEQ I KEY THEN | 00016500 | 009:0007:1 |
| | GO TO L20; | 00016600 | 009:0008:3 |
| | A(IJ) := A(I); | 00016700 | 009:000C:3 |
| | A(I) := T; | 00016800 | 009:000D:0 |
| | T := A(IJ); | 00016900 | 009:000F:4 |
| | T := A(IJ); | 00017000 | 009:0011:3 |
| L20: | L := J; | 00017100 | 009:0013:1 |
| | IF A(IJ) KEY GEQ T KEY THEN | 00017200 | 009:0013:1 |
| | GO TO L40; | 00017300 | 009:0014:1 |
| | A(IJ) := A(I); | 00017400 | 009:0018:1 |
| | A(I) := T; | 00017500 | 009:0018:4 |
| | T := A(IJ); | 00017600 | 009:001B:2 |
| | IF ALL KEY LEQ I KEY THEN | 00017700 | 009:001D:1 |
| | GO TO L40; | 00017800 | 009:001E:5 |
| | A(IJ) := A(I); | 00017900 | 009:0022:5 |
| | A(I) := T; | 00018000 | 009:0023:2 |
| | T := A(IJ); | 00018100 | 009:0026:0 |
| | T := A(IJ); | 00018200 | 009:0027:5 |
| | GO TO L40; | 00018300 | 009:0029:3 |
| L30: | A(I) := A(K); | 00018400 | 009:002A:0 |
| | A(K) := T; | 00018500 | 009:002A:0 |
| | | 00018600 | 009:002C:4 |
| L40: | L := L-1; | 00018700 | 009:002E:3 |
| | IF A(IJ) KEY GTR T KEY THEN | 00018800 | 009:002E:3 |
| | GO TO L40; | 00018900 | 009:002F:5 |
| | TT := A(IJ); | 00019000 | 009:0033:3 |
| | | 00019100 | 009:0034:0 |
| L50: | K := K+1; | 00019200 | 009:0035:4 |
| | IF A(K) KEY LSS I KEY THEN | 00019300 | 009:0035:4 |
| | GO TO L60; | 00019400 | 009:0037:0 |
| | IF K LEQ L THEN | 00019500 | 009:003A:4 |
| | GO TO L30; | 00019600 | 009:003B:1 |
| | IF (I-I) LEQ (J-K) THEN | 00019700 | 009:003C:0 |
| | GO TO L60; | 00019800 | 009:003C:3 |
| | I(I) := I; | 00019900 | 009:003E:2 |
| | I(U) := L; | 00020000 | 009:003E:5 |
| | I := K; | 00020100 | 009:0040:1 |
| | M := M+1; | 00020200 | 009:0041:3 |
| | GO TO L80; | 00020300 | 009:0042:3 |
| L60: | I(I) := K; | 00020400 | 009:0043:5 |
| | I(U) := J; | 00020500 | 009:0044:2 |
| | J := L; | 00020600 | 009:0044:2 |
| | M := M+1; | 00020700 | 009:0045:4 |
| | GO TO L80; | 00020800 | 009:0047:0 |
| L70: | M := M-1; | 00020900 | 009:0048:0 |
| | IF M=0 THEN | 00021000 | 009:0049:5 |
| | GO TO EXIT; | 00021100 | 009:0049:5 |
| | I := I(I); | 00021200 | 009:0049:5 |
| | J := I(U); | 00021300 | 009:004B:1 |
| | | 00021400 | 009:004B:3 |
| L80: | IF (J-I) GEQ 1 THEN | 00021500 | 009:004C:0 |
| | GO TO L10; | 00021600 | 009:004D:2 |
| | IF I EQL IB THEN | 00021700 | 009:004E:4 |
| | | 00021800 | 009:004E:4 |
| | | 00021900 | 009:004F:5 |
| | | 00022000 | 009:0050:2 |

| | | |
|----------------------------|--------------------------|------------|
| GO TO L5; | 00022100 | 009:0051:1 |
| I := I-1; | 00022200 | 009:0051:4 |
| | 00022300 | 009:0053:0 |
| I := I+1; | 00022400 | 009:0053:0 |
| IF I EQL J THEN | 00022500 | 009:0054:2 |
| GO TO L70; | 00022600 | 009:0054:5 |
| T := A(I+1); | 00022700 | 009:0055:2 |
| IF A(I).KEY LEQ T.KEY THEN | 00022800 | 009:0057:2 |
| GO TO L90; | 00022900 | 009:0058:2 |
| K := I; | 00023000 | 009:0058:5 |
| | 00023100 | 009:0058:5 |
| A(K+1) := A(K); | 00023200 | 009:0058:5 |
| K := K-1; | 00023300 | 009:005F:5 |
| IF T.KEY LSS A(K).KEY THEN | 00023400 | 009:0061:1 |
| GO TO L100; | 00023500 | 009:0063:1 |
| A(K+1) := T; | 00023600 | 009:0063:4 |
| GO TO L90; | 00023700 | 009:0067:5 |
| | 00023800 | 009:0068:2 |
| END OF SORT ROUTINE; | 00023900 | 009:0068:2 |
| | 60RTER(009) IS 0070 LONG | |

EXIT;

```

BIGH := 99999;
READLN(I1,I2,NCOLS,NJOBS);
FOR I1:=1 STEP 1 UNTIL NCOLS-1 DO
  TOTJOBS := *NJOBS(I1);
  KUP := NROWS-TOTJOBS;
  NJOBS(I1) := NROWS;
  WRITE(PRIN,/, " AFSC NR. JOBS");
  FOR I2:=0 STEP 1 UNTIL NCOLS-1 DO
    WRITE(PRIN,<X4,16,X6,16>,(I1+1),NJOBS(I1));
  FOR I2:=0 STEP 1 UNTIL NROWS-1 DO
    ID(I1) := I1(I1) := I1(I1) := I1(I1) := 0;
  FOR I2:=NROWS+1 STEP 1 UNTIL NROWS+NCOLS DO
    I1(I1) := -9999;
  FOR I2:=1 STEP 1 UNTIL NCOLS DO
    IDA(I1+NROWS) := NJOBS(I1-1);
    IDA(NCOLS+1) := NROWS;
  FOR I2:=0 STEP 1 UNTIL NROWS-1 DO
    QUANT(I1) := 1;
  INIT := 0;
  UP := NROWS-1;
  START:
  COLLO1 := 0;
  FOR I1:=INIT STEP 1 UNTIL NROWS-1 DO
    IF QUANT(I1)>0 THEN
      BEGIN
        ROWSUM := 0;
        ROWSQ := 0;
        READ(INP1(I1+2),NCOLS,COST);
        TIME3 := TIME(2);
        FOR J1:=0 STEP 1 UNTIL NCOLS-1 DO
          BEGIN
            JOBS := NJOBS(J1);
            IF J1=0 THEN
              KOSIJ := 0
            ELSE
              IF COST(J1)>9999 THEN
                KOSTJ := 1026
              ELSE
                KOSTJ := COST(J1);
            COLSQ(J1) := *(KOSTJ*KOSTJ);
            ROWTEM := KOSTJ*JOBS;
            ROWSUM := *ROWTEM;
            COL(J1) := *KOSTJ*QUANT(I1);
            ROWSQ := *ROWTEM*KOSTJ;
            COST(J1) := KOSTJ;
          END;
        COST(I1) := (ROWSQ-((ROWSUM*2)/((TOTJOBS+0))))/
          (TOTJOBS);
        COST(I1) := SORT(COST(I1));
        COLSI(I1) := ROWSUM/(TOTJOBS*(COST(I1)+1.0000));
        ROWTOT(I1) := COST(I1)*100;
        ROWTOT(I1) := INTEGER(ROWTOT(I1));
        PACK(ROWTOT(I1),I1);
        COLSQ(I1) := *(COST(I1)*COST(I1));
        COL(I1) := *COST(I1);
        IF I LEQ UP THEN
          WRITE(D1JS(I1),NCOLS,COST);
        TIME3 := TIME(2)-TIME3;
        TIME4 := *TIME3;
      END;
    END;
  
```

```

SORTER(ROWTOT,0,NROWS-1,33,34);
FOR I:=0 STEP 1 UNTIL NCOLS-1 DO
  COLSIG(I) :=
    SORT((COLSIG(I)-(COL(I)*COL(I))/NROWS)/NROWS);
FOR I:=0 STEP 1 UNTIL NCOLS-1 DO
  COL(I) := 0;
TIME4 := TIME4/60;
WRITE(PRIN,<F7.2>,TIME4);
TIME4 := 0;
FOR J:=INIT STEP 1 UNTIL UP DO
  BEGIN
    IF TOTJOBS=0 THEN
      GO TO WRAP;
    COUNT := J;
    COUNT := UNPACKJ(ROWTOT(COUNT));
    READ(DIJS(COUNT),NCOLS,COST);
    NM := 0;
    K := 0;
    LL := 0;
    BUF := 10*63;
    FOR I:=0 STEP 1 UNTIL NCOLS-1 DO
      BEGIN
        IF NJOBS(I)>0 THEN
          BEGIN
            LL := *+1;
            DI := COST(I)-COL(I)/(NASSN-J+0.001);
            IF DI<BUF THEN
              BEGIN
                BUF := DI;
                NM := LL;
                K := I;
              END;
            END;
          END;
        I := K;
        DI := BUF;
        IF COST(I)<999 OR I=0 THEN
          BEGIN
            NJOBS(I) := *+1;
            IF I NEQ 0 THEN
              TOTJOBS := *+1;
            COUNT := *+1;
            IF I(NROWS+1) NEQ 0 THEN
              I(COUNT) := I(NROWS+1);
              I(NROWS+1) := COUNT;
            IF I=0 THEN
              I(COUNT) := I(NROWS+1)+999
            ELSE
              I(COUNT) := I(NROWS+1)+COST(I);
            ID(COUNT) := NROWS+1;
            IDA(NROWS+1) := *+1;
            IDA(COUNT) := 1;
            OFFERS(COUNT,1) := 1;
            QUANT(COUNT) := 0;
            Z := COUNT;
            IF I=0 THEN
              OBJVAL := *+999
            ELSE
              OBJVAL := *+COST(I);
            WRITE(PRIN,<F7.6F13.2>,I,IDA(Z),ID(Z),IUI(Z),

```

```

00030900 008:00B5:1
00031000 008:00B0:4
00031100 008:00C1:3
00031200 008:00C2:1
00031300 008:00C7:0
00031400 008:00C8:5
00031500 008:00C0:4
00031600 008:00C0:4
00031700 008:00CF:0
00031800 008:00D6:2
00031900 008:00D7:0
00032000 008:00D7:0
00032100 008:00D8:0
00032200 008:00D8:0
00032300 008:00D8:0
00032400 008:00D8:4
00032500 008:00D0:1
00032600 008:00D0:1
00032700 008:00D0:1
00032800 008:00DE:4
00032900 008:00DE:4
00033000 008:00E6:2
00033100 008:00E7:0
00033200 008:00E7:4
00033300 008:00E7:2
00033400 008:00E8:2
00033500 008:00E8:2
00033600 008:00EF:2
00033700 008:00F0:2
00033800 008:00F0:5
00033900 008:00F2:0
00034000 008:00F7:0
00034100 008:00F7:3
00034200 008:00F7:3
00034300 008:00F8:0
00034400 008:00F8:5
00034500 008:00F9:4
00034600 008:00FA:4
00034700 008:00FA:4
00034800 008:00FA:4
00034900 008:00FA:4
00035000 008:00FA:4
00035100 008:00FB:1
00035200 008:00FC:1
00035300 008:00FD:0
00035400 008:00FE:1
00035500 008:00FF:4
00035600 008:0101:2
00035700 008:0102:0
00035800 008:0103:4
00035900 008:0105:0
00036000 008:0106:5
00036100 008:0109:5
00036200 008:010C:0
00036300 008:010C:4
00036400 008:010F:3
00036500 008:0110:2
00036600 008:0114:1
00036700 008:0116:2
00036800 008:0118:5
00036900 008:011A:0
00037000 008:011B:4
00037100 008:011C:5
00037200 008:011D:5
00037300 008:011E:3
00037400 008:011F:2
00037500 008:0120:3
00037600 008:0122:4

```



```
IR(Z),IP(Z),COUNT,DI,COSI(I),OB,IVAL,COST(I),
COL(I),COLSIG(I));
```

```
COUNT := * - 1;
DEC := * + 1;
```

```
END
ELSE
```

```
EXCESS(I) := * + 1;
FOR KUP := 0 STEP 1 UNTIL NCOLS - 1 DO
  COL(KUP) := * - COST(KUP);
```

```
END;
```

```
TIME1 := TIME(2) - TIME1;
TIME2 := * + TIME1;
TIME1 := TIME1 / 60;
WRITE(PRIN, <F7.2>, TIME1);
```

```
MASSN := * - DEC;
```

```
FOR I := 0 STEP 1 UNTIL NROWS - 1 DO
  ROWTOT(I) := 0;
```

```
FOR I := 0 STEP 1 UNTIL NCOLS - 1 DO
```

```
  COLSIG(I) := COLSIG(I) + COL(I) := 0;
```

```
  INIT := * + UP + 1;
```

```
  UP := NROWS - 1;
```

```
  IF INIT LEQ NROWS - 1 THEN
```

```
    GO TO START;
```

```
*****
```

```
WRAP:
```

```
I := 0;
```

```
FOR COUNT := 1 STEP 1 UNTIL NROWS DO
```

```
  BEGIN
```

```
    IF QUANT(COUNT) = 1 THEN
```

```
      BEGIN
```

```
        IF I(NROWS + I + 1) NEQ 0 THEN
```

```
          I(COUNT) := I(NROWS + I + 1);
```

```
          I(NROWS + I + 1) := COUNT;
```

```
          IF I = 0 THEN
```

```
            I(COUNT) := I(NROWS + I + 1) + 999
```

```
          ELSE
```

```
            I(COUNT) := I(NROWS + I + 1) + COST(COUNT);
```

```
            I(COUNT) := NROWS + I + 1;
```

```
            I(NROWS + I + 1) := * - 1;
```

```
            I(COUNT) := 1;
```

```
            OFFERS(COUNT, I) := 1;
```

```
            QUANT(COUNT) := 0;
```

```
            OBJVAL := * + 999;
```

```
          END;
```

```
        END;
```

```
      WRITE(PRIN, /, " AFSC EXCESS JOBS");
```

```
*****
```

```
      FOR I := 0 STEP 1 UNTIL NCOLS - 1 DO
```

```
        WRITE(PRIN, F6, (I + 1), NJOBS(I));
```

```
        WRITE(PRIN, <" OBJECTIVE FUNCTION VALUE= ", F10.0>,
```

```
        OBJVAL);
```

```
      X FOR I := 1 STEP 1 UNTIL NCOLS DO WRITE(PRIN, F5, I, (OFFERS(I, I + 1)),
```

```
      X OFFERS(I, 2), OFFERS(I, 3));
```

```
      X WRITE(PRIN, /, " AFSC EXCESS JOBS");
```

```
      X FOR I := 1 STEP 1 UNTIL NROWS - 1 DO WRITE(PRIN, F8, (I + 1), EXCESS(I));
```

```
      X FOR I := 1 STEP 1 UNTIL NCOLS DO
```

```
        FOR I := 1 STEP 1 UNTIL NCOLS DO
```

```
          WRITE(OTP, <2I6>, I, (OFFERS(I, I + 1)));
```

```
      X WRITE(OTP, <2I6>, I, (OFFERS(I, I + 1)));
```

```
00037700 008:0120:5
```

```
00037800 008:013A:2
```

```
00037900 008:0141:2
```

```
00038000 008:0142:4
```

```
00038100 008:0143:5
```

```
00038200 008:0143:5
```

```
00038300 008:0143:5
```

```
00038400 008:0146:0
```

```
00038500 008:014A:5
```

```
00038600 008:014D:3
```

```
00038700 008:014E:0
```

```
00038800 008:0150:0
```

```
00038900 008:0151:2
```

```
00039000 008:0152:4
```

```
00039100 008:015A:2
```

```
00039200 008:015B:4
```

```
00039300 008:0160:3
```

```
00039400 008:0162:1
```

```
00039500 008:0167:0
```

```
00039700 008:016A:4
```

```
00039800 008:016C:3
```

```
00039900 008:016D:5
```

```
00040000 008:016F:0
```

```
00040100 008:016F:3
```

```
00040200 008:016F:3
```

```
00040300 008:016F:3
```

```
00040400 008:0170:1
```

```
00040500 008:0174:4
```

```
00040600 008:0174:4
```

```
00040700 008:0175:4
```

```
00040800 008:0176:1
```

```
00040900 008:0178:0
```

```
00041000 008:0178:0
```

```
00041100 008:017D:1
```

```
00041200 008:017D:5
```

```
00041300 008:0180:4
```

```
00041400 008:0181:3
```

```
00041500 008:0185:2
```

```
00041600 008:0187:3
```

```
00041700 008:018A:0
```

```
00041800 008:018B:1
```

```
00041900 008:018C:5
```

```
00042000 008:018E:0
```

```
00042100 008:018F:3
```

```
00042200 008:018F:3
```

```
00042300 008:0190:0
```

```
00042400 008:0199:2
```

```
00042500 008:0199:2
```

```
00042600 008:019E:1
```

```
00042700 008:01A8:5
```

```
DATA 18 0027 LONG
```

```
00042800 008:01AB:3
```

```
00042900 008:01B0:2
```

```
00043000 008:01B0:2
```

```
00043100 008:01B0:2
```

```
00043200 008:01B0:2
```

```
00043300 008:01B0:2
```

```
00043400 008:01B0:2
```

```
00043500 008:01B4:5
```

```
00043600 008:01BE:5
```

```

X WRITE(PRI, <14,416,110>,FOR I:=1 STEP 1 UNTIL NROWS+NCOLS DO
X   [I,IDA(I),ID(I),IUL(I),IR(I),IP(I)]);
NROWS := NROWS+NCOLS;

X ADVAN.MAXRECSIZE:=5*NROWS;
X ADVAN.BLOCKSIZE:=5*NROWS;

ADVAN.MAXRECSIZE := 4*NROWS;
ADVAN.BLOCKSIZE := 4*NROWS;
WRITE(ADVAN,*,FOR I:=1 STEP 1 UNTIL NROWS
DO[I,IDA(I),ID(I),IR(I),IP(I)]);
X   [I,IDA(I),ID(I),IR(I),IP(I)]);
LOCK(OTP);
LOCK(ADVAN);
TIMER := TIME2 DIV 60;
WRITE(PRI,*,TIME2,TIME);
END;

END.

=====
NUMBER OF ERRORS DETECTED = 0.
NUMBER OF SEGMENTS = 11. TOTAL SEGMENT SIZE = 831 WORDS. CORE ESTIMATE = 3670 WORDS. STACK ESTIMATE = 99
PROGRAM SIZE = 434 CARDS, 2291 SYNTACTIC ITEMS, 77 DISK SEGMENTS.
PROGRAM FILE NAME: (09DA00)BEST.
COMPILATION TIME = 24.280 SECONDS ELAPSED; 0.363 SECONDS PROCESSING; 2.697 SECONDS I/O.
=====
DATA IS 0010 LONG
=====

```